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First Order Ball Bearing Kinematics

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ABSTRACT

Two first order equations are given connecting geometry and internal motions in an angular contact ball bearing. Total speed, kinematic equivalence, basic speed ratio, and modal speed ratio are defined and discussed; charts are given for the speed ratios covering all bearings and all rotational modes. Instances where specific first order assumptions might fail are discussed, and the resulting effects on bearing performance reviewed.

INTRODUCTION

First order kinematic analysis is concerned with the relations between the sizes and the motions of balls and races in a bearing. The equations relating race rotations to ball and ball group rotations ("speed ratios") are usually given separately for the two specific cases were one or the other race is stationary - four equations for these two special cases (1). There are an infinite number of other combinations for race rotation where these formulas fail. A more general treatment has been given (2, 3), but in a rather complicated form which obscures somewhat the simple underlying physical situation.

All these theories are first order, implicitly or explicitly invoking five assumptions, with the idea that a first order treatment will give a useful foundation for bearing analysis. Complications may then be added to

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the theory one at a time as appears necessary. The assumptions are: (a) rigid balls and races, (b) zero circumferential ball-race slip, (c) equal inner and outer contact angles, (d) no rolling friction, and (e) no acceleration of the bearing center of mass.

In this paper it is shown that, under the first order assumptions, two very general but very simple formulas are adequate to describe all possible angular contact ball bearings for all possible combinations of race rotation. Moreover the variables in the equations are natural and intuitive; their physical meaning makes the equations easy to remember and easy to apply.

With this as background, the application of accurate rate measurement to first order kinematic analysis is then considered to show how information on specific dynamical and rheological bearing effects can be obtained.

GEOMETRY

A bearing can be regarded as a group of balls arranged in a circle, rolling under load between toroidal inner and outer races. The proportions of the bearing are generalized by D, the ratio of ball to ball group (pitch) diameter: $D = d/E$. A bearing has many "small" balls if $d \ll E$. D can vary from zero in a linear bearing $(E = \infty)$ to 1 in a bearing with two large balls.

Bearing loads are carried from race to race across each ball, along lines of contact which pass through each ball center. The angle B between a line of contacts and a normal to the bearing axis is called the contact angle and determines bearing type. B can vary from zero in a radial bearing to 90 degrees in a thrust bearing (Fig. 1).

D and B form a pair of independent bearing variables which completely describe first order geometry. Either can take any value over its range, independent of the other (except that a bearing with $D = 1$ and $B = 0$ can

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have no inner race). Fig. 2 shows a two-ball bearing for $B = 0$ and $B = 90$ degrees.

Some derived geometrics are a, the angle around the pitch circle between two adjacent balls in a full complement bearing:

$$a = 2 \sin^{-1}(D) \qquad (1)$$

N, the maximum number of balls in a full complement:

$$N = \text{int} (2\pi/a) \qquad (2)$$

and G, the angular gap outside a full complement:

$$G = 2\pi - Na \qquad (3)$$

Fig. 3 is a plot of N against D. N takes on integral values, jumping to smaller values as D increases. $G = a$ at each jump and decreases to zero just before the next jump.

ROTATIONS

A ball bearing has four angular rates:

$\dot{\gamma}_0$	outer race
$\dot{\gamma}_I$	inner race
$\dot{\beta}$	ball group orbit
$\dot{\delta}$	ball-about-its-own-center (spin)

The inner and outer rates are another pair of independent bearing variables. Either can take any value over the range $\pm\infty$; each combination is called a "mode". In practice one or the other rate is usually zero, hence the terms "inner or outer race rotation," but there is no theoretical requirement that one race be stationary.

KINEMATICS

For arbitrary race rotations define the total speed of the bearing as

$$S = \dot{\gamma}_0 - \dot{\gamma}_I \tag{4}$$

and the average speed as

$$A = (\dot{\gamma}_0 + \dot{\gamma}_I)/2 \tag{5}$$

taking account of the signs of the rotations.

Then, for the first order assumptions

$$\dot{\delta} = S\,(1/2D)\,(1 - D^2 \cos^2 B) \tag{6}$$

and

$$\dot{\beta} = A + S\,(D \cos B)/2 \tag{7}$$

These are the general first order kinematic equations connecting geometry and rotation in any bearing. Ball spin rate depends only on geometry and total speed; ball orbit rate is the average speed corrected by a geometrical fraction of total speed. The significance of S and A is seen from a comparison of a bearing running in two special modes:

I	II
$\dot{\gamma}_0 = -\dot{\gamma}_I = \dot{\gamma}$	$\dot{\gamma}_0 = \dot{\gamma}_I = \dot{\gamma}$
$S = 2\dot{\gamma}$	$S = 0$
$A = 0$	$A = \dot{\gamma}$
$\dot{\delta} = 2\,\dot{\gamma}\,(1/2\,D)\,(1 - D^2 \cos^2 B)$	$\dot{\delta} = 0$
$\dot{\beta} = \dot{\gamma}\,(D \cos B)$	$\dot{\beta} = \dot{\gamma}$

Total speed thus measures the relative rotation inside a bearing; average speed is a measure of its solid body rotation.

Define the basic speed ratio of any bearing as

$$\rho = \dot{\delta}/S = (1/2 \ D) \ (1 - D^2 \cos^2 B) \tag{8}$$

ρ is called basic because it depends only on geometry, not on mode. It is a characteristic number for an angular contact ball bearing.

Define modal speed ratio as

$$\rho_M = \dot{\beta}/S = A/S + (D \cos B)/2 \tag{9}$$

ρ_M is the sum of a modal term, A/S, and a geometric correction whose maximum value could be 1/2. Both ρ and ρ_M are defined for all modes except at S = 0 (Mode II). But in II we really have a wheel, not a bearing at all.

Thus the kinematics of all bearings in all modes can be described with two diagrams, one for ρ and one for ρ_M.

Fig. 4 is such a diagram; ρ vs B showing lines of constant D. The number of balls for a full complement at each D is also given (from Fig. 3). Only at large D is there any appreciable effect of contact angle. For D < 0.1 an approximation good to 1 percent is given by $\rho = 1/(2D)$. The chart of ρ vs B extends to ∞ as D approaches 0; however most bearings have ρ < 10, N < 62, and are included in Fig. 4. In practice ρ increases in a general way with bearing size. A typical value for a small instrument bearing is 2, for an aircraft gas turbine bearing 4, and for a crane turntable bearing 10. Certain specialty bearings (torque tube) may have ρ as large as 50, and contain over 300 balls.

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Fig. 5 is a plot of the modal part of ρ_M vs $\dot{\gamma}_I/\dot{\gamma}_0$, with the point for outer race rotation marked. Both branches are asymptotic to –1/2, one from above, the other from below. Inner race rotation is at $\dot{\gamma}_I/\dot{\gamma}_0 = \infty$ where $A/S = -1/2$. To find ρ_M from Fig. 5 it is only necessary to add the geometric correction $(D \cos B)/2$ to the A/S of interest. The correction is always positive, approaching zero for small balls or thrust loading, and 1/2 for large balls or radial loading.

As an example of the use of Fig. 5, consider bearing modes with zero ball orbit rate (there is a theoretical interest in these modes since ball centrifugal force is then zero). Eq. (9) shows that A/S must be between 0 and –1/2 for $\dot{\beta} = 0$. In Fig. 5 this region, lying to the left of $\dot{\gamma}_I/\dot{\gamma}_0 = -1$, is shown shaded. Only here could a suitable choice of D and B give $\dot{\beta} = 0$. Thus the race rotations must be opposite, giving the name "counter race rotation" to these modes.

APPLICATIONS

Basic Speed Ratio

Basic speed ratio can be measured extremely accurately in a bearing run in the counter race rotation mode at zero ball orbit rate. For stationary ball centers individual spin rates can easily be obtained to 1 part in 10^5, and ρ computed from its definition, Eq. (8). Since, to first order ρ is a geometric constant, any measured change reflects some deviation from the first order assumptions. This is useful in determining running conditions where specific assumptions might break down, and in evaluating numerical models of bearing performance (4).

In a successful bearing balls and races are separated by an elastohydro-dynamic lubricant film. Since ball spin is driven by traction forces generated in this film, ρ gives a sensitive evaluation of ball-lubricant-race

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coupling. Relative changes in film thickness (5) and absolute values for inner and outer ball-race circumferential slip (6) have been obtained from ρ measurements.

To find circumferential slip it is necessary to know the geometry of the test bearing and its rotation rates to a comparable accuracy. Direct measurement (e.g. with a micrometer) at this accuracy is possible for ball diameter, but impossible for pitch diameter and contact angle. However it turns out for no circumferential slip in counter rotation (6)

$$\cos B = \begin{bmatrix} \dot{\delta}/2 & \dot{\gamma}_0 & \dot{\gamma}_I \end{bmatrix} \begin{bmatrix} (\dot{\gamma}_I + \dot{\gamma}_0) \end{bmatrix} \tag{10}$$

$$E = \begin{bmatrix} \dot{\delta}/2 & \dot{\gamma}_0 \dot{\gamma}_I \end{bmatrix} \begin{bmatrix} (\dot{\gamma}_I - \dot{\gamma}_0) \end{bmatrix} d \tag{11}$$

Thus E and B can be accurately determined by means of a small slip experiment (e.g., low total speed, thin film) and then used to find circumferential slip at high speed and thick film. This particular slip depends among other things on lubricant rheology and film thickness. Experimental information on changes in film configuration can thus easily be obtained in a real bearing under realistic operating conditions.

Total Speed

Total speed measures the relative rotation inside a bearing, giving a basis for comparing the severity of different operating modes. Since ball spin rate is the product of ρ and total speed it is the same for all modes which have the same total speed. The details of the approach, contact and retreat of load carrying elements on ball and race are the same for all modes at the same total speed. To first order "all modes at the same total speed are kinematically equivalent". Classical calculation of elastohydrodynamic film thickness depends on these kinematic details.

It can also be shown (7) that the total inner and outer ball-race pivoting (the components of relative angular velocity normal to each Hertz area, often loosely referred to as spin), is proportional to total speed for all positions of the ball spin vector:

$$P_0 + P_I = S \sin B \tag{12}$$

Normalizing with respect to ball spin rate

$$\Sigma P / \dot{\delta} = \sin B / \rho \tag{13}$$

a geometric constant independent of mode, speed, or change in ball spin orientation. The ball-race slip associated with pivoting is "kinematic," dictated solely by bearing geometry and total speed. It can never be zero except in a pure radial bearing. In particular, pivoting slip is completely independent of lubricant rheology and film thickness; in a sense just the opposite of circumferential slip.

Although lubrication does not affect pivoting slip, the reverse is not true. It has been suggested (8) that lubricant shear associated with pivoting slip activates polymerization or oxidation reactions in organic lubricants. In a starved bearing rapid lubricant failure may result if the elastohydrodynamic film becomes too thin, even though it is still complete (unpenetrated).

Modal Speed Ratio

Kinematic equivalence means that all modes at the same total speed would look the same to an observer orbiting with the ball group. This is only true as long as the centrifugal force on a ball due to $\dot{\beta}$ is small compared to the service load across the ball. If the total normal force at the ball-outer contact were significantly larger than that at the inner, the observer would measure a decreased outer contact angle; the line of contacts would not go through the ball center, and a first order assumption would be violated. Ball centrifugal force c depends on ball orbit rate squared; the latter is easily

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computed from Eq. (9), given a definite mode, total speed, and bearing
geometry.

A reduced outer angle is given by

$$B_0 = \tan^{-1} (\sin B_I / (\cos B_I + c/n))$$ (14)

Fig. 6 plots $(B_I - B_0)$ vs B_I for different values of c/n, cen-
trifugal divided by inner normal ball force. The difference is less than 6
degrees in all bearings if c is no more than 10 percent of the service
load. The effect becomes less important in general as bearings become more
"radial"; for contact angles less than $30°$ the difference is less than
10 degrees for centrifugal forces as much as half the service load. Ball
centrifugal force is proportional to $\dot{\beta}^2$, d^3, and E. First order
analysis becomes suspect for large bearings and/or high average speeds (Eq. 7).

Another dynamic effect proportional to $\dot{\beta}^2$ arises in a full complement
bearing with a ball group residual gap (Eq. (3)). Then the ball group center
of mass does not lie on the bearing spin axis and there will be a radial group
unbalance force rotating around the bearing axis at ball orbit rate. A
similar force results from the eccentric whirl motion of the cage in a
conventional bearing. This force cannot be removed by ordinary balancing
since $\dot{\beta}$ is never the same as either race rate (except in mode II or in
certain unusual special cases). The forces from two bearings can combine to
produce a variable torque normal to the spin axis of the pair (9); mechanical
"crosstorque" is undesirable in gyroscope and satellite applications.

In a full complement bearing it can be shown (for D small enough so
$D \simeq \sin^{-1} (D)$, say less than 0.5) that the radius from the bearing axis
to the ball group center of mass is

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$$r = (N + 1)e/2 \tag{15}$$

where e is the difference between the actual ball diameter and the next larger diameter for zero gap $d_{G=0}$:

$$e = d_{G=0} - d \tag{16}$$

Thus if a bearing has 10 balls $(D = 0.3)$ it makes no difference if it has a pitch diameter of one mm or one km: when the ball diameter is reduced 0.001 mm from that required for zero gap the center of mass of the ball group will be located 0.00175 mm radially from the bearing axis in either case.

CONCLUDING REMARKS

In its first order form an angular contact bearing is a very simple device. When it is run in the counter rotating mode under a pure axial load very accurate kinematic measurements can be made easily. Together these features make the analysis of real bearings in real operating situations feasible, interesting, and occasionally surprising. The kinematic, dynamic, rheologic, and chemical effects reviewed here are not the only facets of bearing operation which can be studied. These techniques thus offer an alternative to numerical modeling in bearing research.

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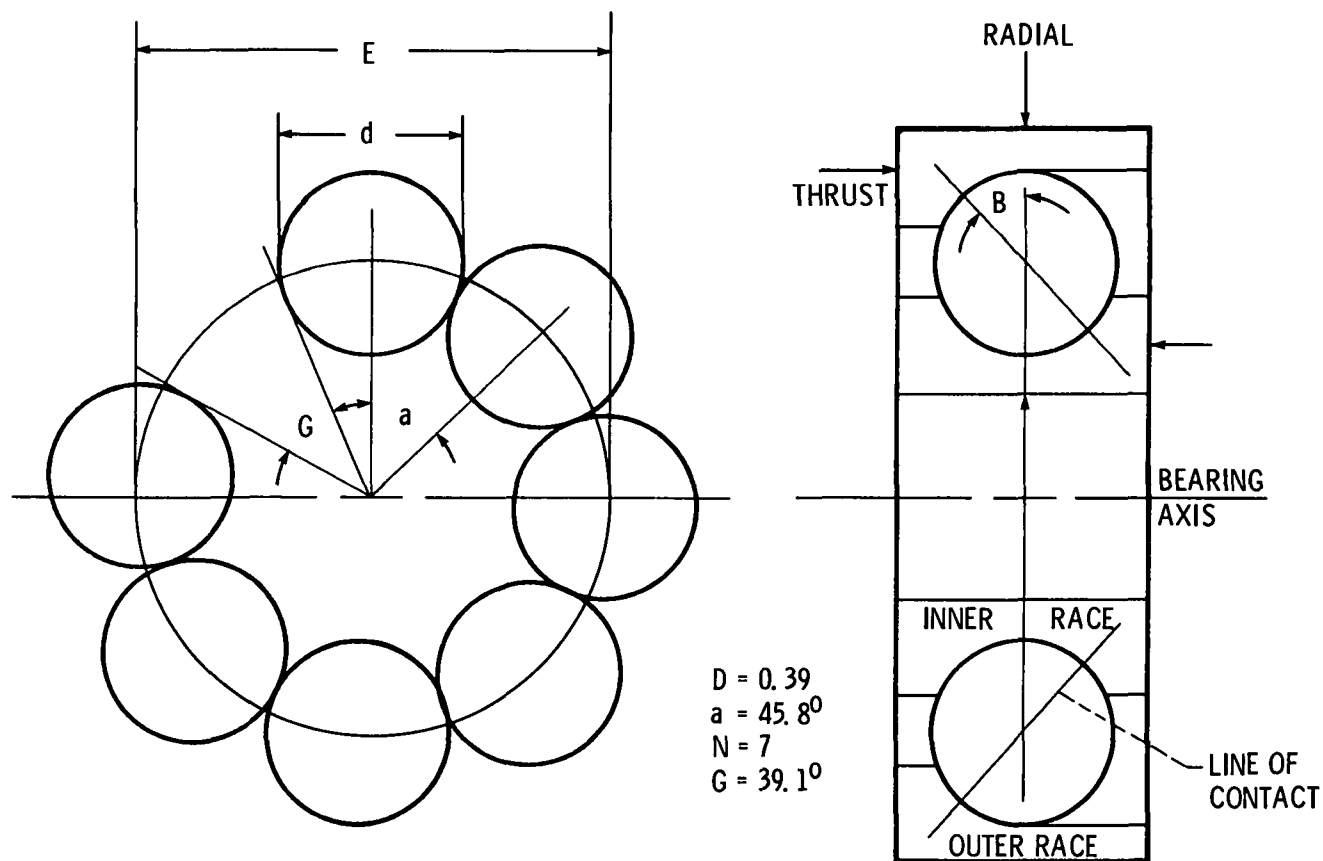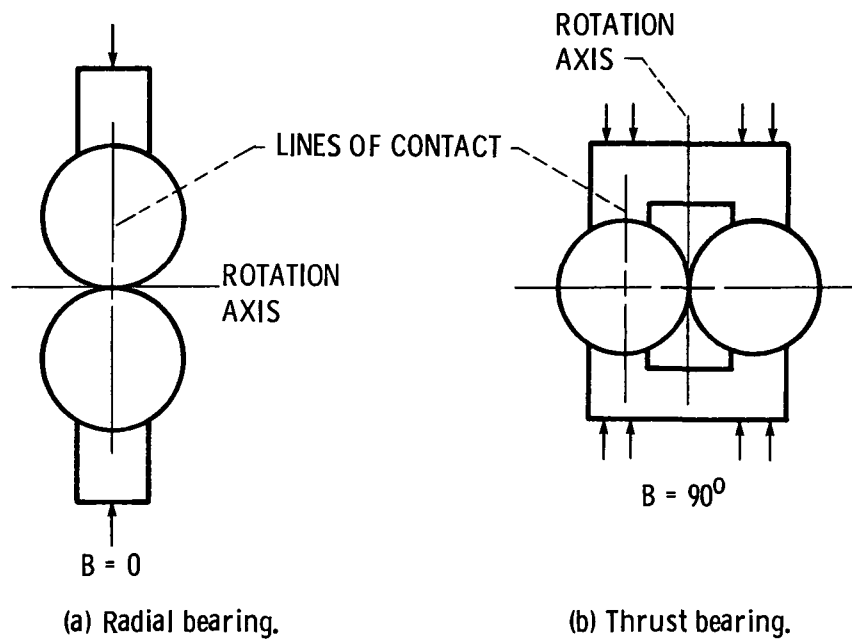
Fig. 1. - Angular contact ball bearing.

(a) Radial bearing. (b) Thrust bearing.

Fig. 2. – Radial and thrust bearings with the largest (D = 1) and fewest (N = 2) balls.
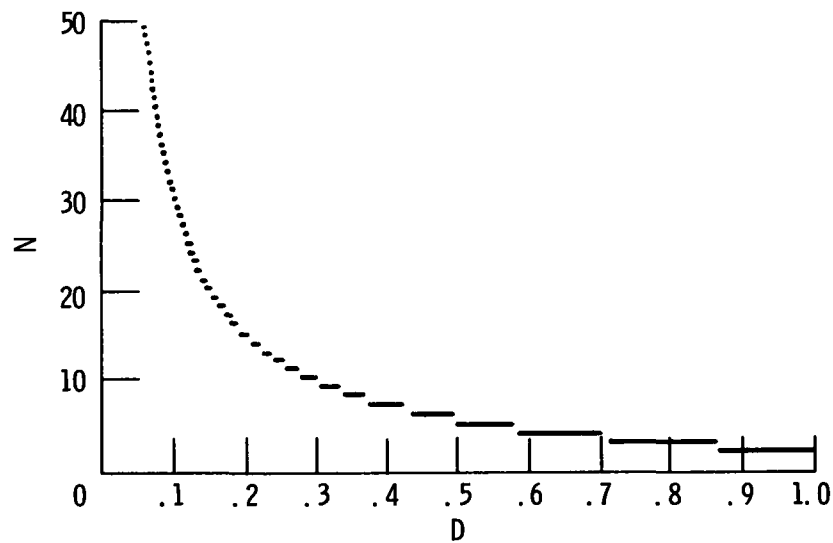
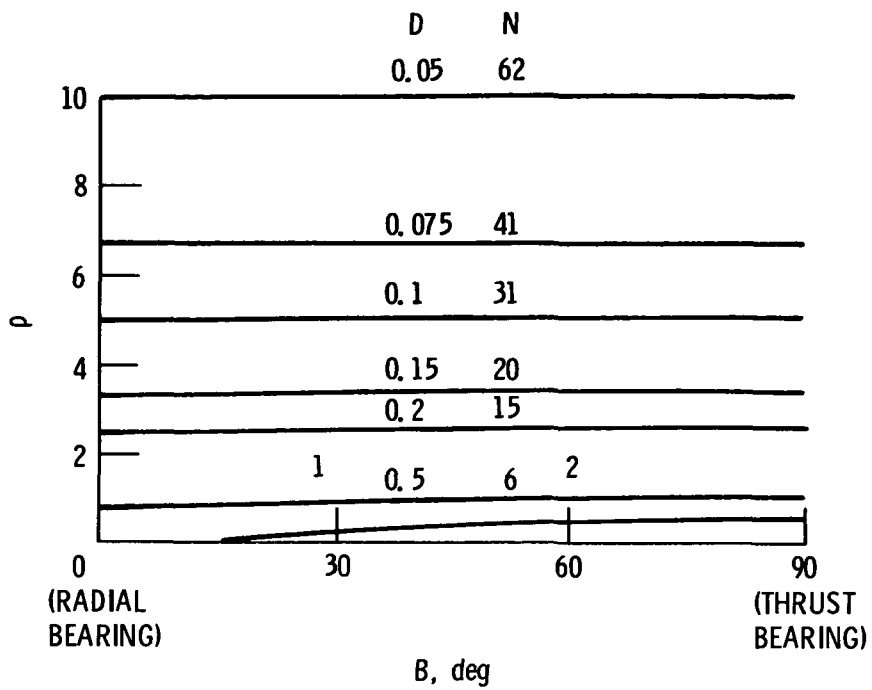Fig. 3. –Maximum ball complement versus ball diameter to pitch diameter ratio.
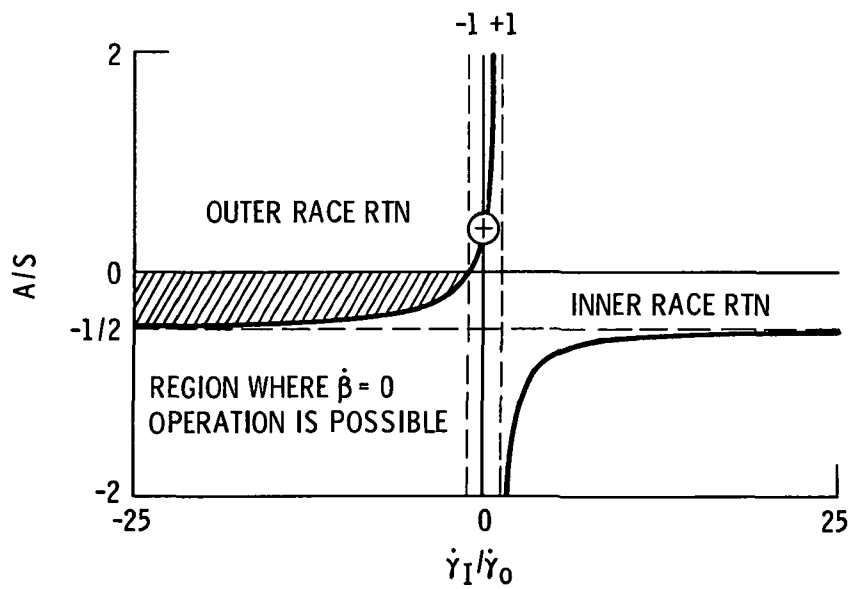
Fig. 4. – Basic speed ratio versus contact angle.

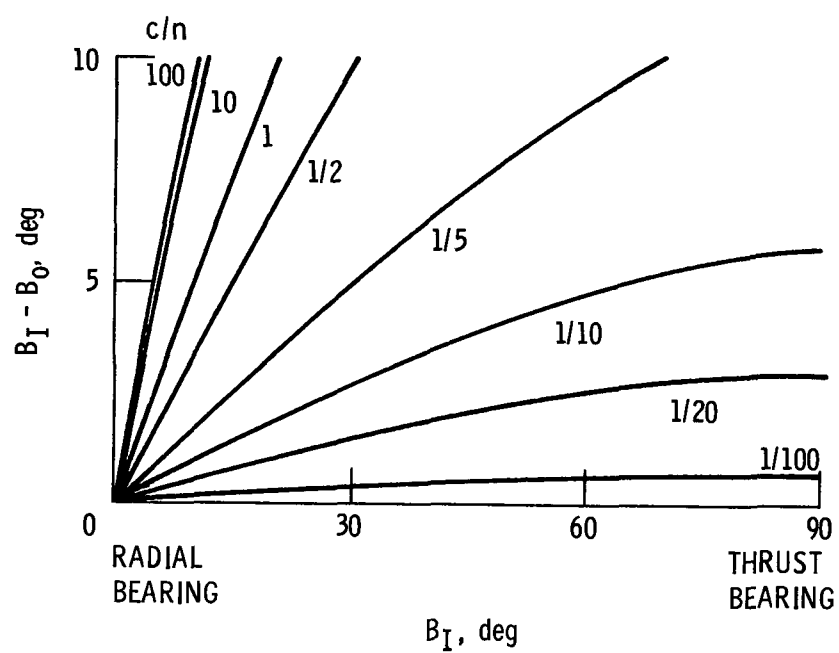Fig. 5. – Modal part of ρ_M (Eq. (9)) versus [inner/outer] rate.

Fig. 6. – Contact angle difference versus inner contact angle.

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